

Mesonic Tensor Form Factors with Light Front Quark Model

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Abstract

We study the tensor form factors for $P \rightarrow P$ and $P \rightarrow V$ transitions in the light-front quark model with P and V being pseudoscalar and vector mesons, respectively. We explore the behaviors of these form factors in the entire physical range of $0 \leq p^2 \leq (M_i - M_f)^2$. At the maximum recoil of $p^2 = 0$, we compare our results of the form factors in $B \rightarrow \pi, K, \rho, K^*$ with various other calculations in the literature.

1 Introduction

The studies of the heavy to light exclusive decays in the standard model are very interesting topics since they can provide the signal of CP violation and a window into new physics. Meanwhile, the calculations of these decay widths with the model-independent way are difficult tasks because they are related to the non-perturbative QCD effects. These effects correspond to the binding of quarks in hadrons and appear in matrix elements of the weak Hamiltonian operators between initial and final hadronic states. However, they could be described by form factors from the parametrization of those hadronic matrix elements in Lorentz invariant. Phenomenologically, these form factors can be evaluated in many different approaches, such as the lattice QCD, the QCD sum rule, the non-relativistic quark model and the light-front quark model (LFQM).

It is well known that when the momentum of the final state meson increases ($> 1\text{GeV}$), we have to consider the relativistic effect seriously, especially, at the maximum recoil of $p^2 = 0$ where the final meson could be highly relativistic, and there is no reason to expect that the non-relativistic quark model is still applicable. While the lattice QCD is still not practically useful for a calculation in hadron physics, the LFQM [1, 2, 3] is the only relativistic quark model in which quark spins and the center-of-mass motion can be carried out in a consistent and fully relativistic treatment. The light-front wave function is manifestly boost invariant as it is expressed in terms of the longitudinal momentum fraction and relative transverse momentum variables. The parameter in the hadronic wave function is determined from other information and the meson state of the definite spins could be constructed by the Melosh transformation [4].

The LFQM has been widely applied to study the form factors of weak decays [5, 6, 7, 8, 9, 10, 11]. In the most pervious works with the LFQM [6, 11], the $P \rightarrow P$ and $P \rightarrow V$ (P =pseudoscalar meson, V =vector meson) tensor form factors were calculated only for $p^2 \leq 0$. However, physical decays occur in the timelike region $0 \leq p^2 \leq (M_i - M_f)^2$ with $M_{i,j}$ being the initial and final meson masses, respectively. Hence, to extrapolate the form factors to cover the entire range of the momentum transfer, some assumptions are needed. For example, in Ref. [12], a light front ansatz for the p^2 dependence was made to extrapolate the form factors in the space-like to the time-like regions and in Ref. [13], based on the dispersion formulation, form factors at $p^2 > 0$ were obtained by performing an analytic continuation from the space-like p^2 region. In Refs. [7, 14, 15], for the first time the form factors in the $P \rightarrow P$ transition were calculated for the entire range of p^2

in the LFQM. In this work, we calculate the tensor form factors for both $P \rightarrow P$ and $P \rightarrow V$ transitions in the entire range of the momentum transfer $0 \leq p^2 \leq (M_i - M_f)^2$. These tensor form factors play an important role in estimating the B decay rates such as that of $B \rightarrow K^{(*)}l^+l^-$ [16].

This paper is organized as follows. In Sec. 2, we study the tensor form factors of $P \rightarrow P$ and $P \rightarrow V$ transitions within the framework of the LFQM. In Sec. 3, we present our numerical results. We give the conclusions in Sec. 4, .

2 Tensor Form Factors

2.1 Framework

We start with the weak tensor operator, given by:

$$O^{\mu\nu} = \bar{q}i\sigma^{\mu\nu}p_\nu(1 + \gamma_5)Q \quad (1)$$

where \bar{q} is the light anti-quark. Our main task is to evaluate the tensor form factors for $P \rightarrow P$ and $P \rightarrow V$ transitions. They are defined by the following hadronic matrix elements:

$$\begin{aligned} \langle P_2(P_f) | \bar{q}i\sigma^{\mu\nu}p_\nu Q | P_1(P_i) \rangle &= \frac{F_T(p^2)}{M_{P_1} + M_{P_2}} [(P_i + P_f)^\mu p^2 - (M_{P_1}^2 - M_{P_2}^2)p^\mu], \\ \langle V(P_f, \epsilon) | \bar{q}i\sigma^{\mu\nu}p_\nu Q | P(P_i) \rangle &= -i\varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* P_{f\alpha} P_{i\beta} F_1(p^2), \\ \langle V(P_f, \epsilon) | \bar{q}i\sigma^{\mu\nu}p_\nu \gamma_5 Q | P(P_i) \rangle &= \left[(M_P^2 - M_V^2)\epsilon^{*\mu} - (\epsilon^* \cdot p)(P_f + P_i)^\mu \right] F_2 \\ &\quad + \epsilon^* \cdot p \left[p^\mu - \frac{p^2}{M_P^2 - M_V^2}(P_f + P_i)^\mu \right] F_3(p^2), \end{aligned} \quad (2)$$

where $P_{i(f)}$ is the momentum of the initial (final) state meson, ϵ is the meson polarization vector and $p = P_i - P_f$.

In calculations of the hadronic matrix elements, one usually lets $p^+ = 0$ which leads to a spacelike momentum transfer. However, since the momentum transfer should be always timelike in a real decay process, in this work, the tensor form factors in Eq. (2) will be calculated in the frame of $p_\perp = 0$, namely the physically accessible kinematic region $0 \leq p^2 \leq p_{\max}^2$. In the LQFM, the meson bound state, which consists of a quark q_1 and an anti-quark \bar{q}_2 with the total momentum P and spin S , can be written as

$$\begin{aligned} |M(P, S, S_z)\rangle &= \int [dk_1][dk_2] 2(2\pi)^3 \delta^3(P - k_1 - k_2) \\ &\quad \times \sum_{\lambda_1 \lambda_2} \Phi^{SS_z}(k_1, k_2, \lambda_1, \lambda_2) |q_1(k_1, \lambda_1) \bar{q}_2(k_2, \lambda_2)\rangle, \end{aligned} \quad (3)$$

where

$$\begin{aligned}
[dk] &= \frac{dk^+ d^2 k_\perp}{2(2\pi)^3}, \\
|q_1(k_1, \lambda_1) \bar{q}_2(k_2, \lambda_2)\rangle &= b_{q_1}^\dagger(k_1, \lambda_1) d_{\bar{q}_2}^\dagger(k_2, \lambda_2) |0\rangle, \\
\{b_{\lambda'}(k'), b_\lambda^\dagger(k)\} &= \{d_{\lambda'}(k'), d_\lambda^\dagger(k)\} = 2(2\pi)^3 \delta^3(k' - k) \delta_{\lambda'\lambda}.
\end{aligned} \tag{4}$$

and $k_{1(2)}$ is the on-mass shell light front momentum of the internal quark $q_1(\bar{q}_2)$. The light-front relative momentum variables (x, k_\perp) are defined by

$$\begin{aligned}
k_1^+ &= x_1 P^+, \quad k_2^+ = x_2 P^+, \quad x_1 + x_2 = 1, \\
k_{1\perp} &= x_1 P_\perp + k_\perp, \quad k_{2\perp} = x_2 P_\perp - k_\perp,
\end{aligned} \tag{5}$$

In the momentum space, the wave function Φ^{SS_z} can be written as:

$$\Phi^{SS_z}(k_1, k_2, \lambda_1, \lambda_2) = R_{\lambda_1 \lambda_2}^{SS_z}(x_i, k_\perp) \phi(x_i, k_\perp), \tag{6}$$

where $\phi(x, k_\perp)$ describes the momentum distribution amplitude of the constituents in the bound state, $R_{\lambda_1 \lambda_2}^{SS_z}$ constructs a spin state (S, S_z) out of light-front helicity eigenstates $(\lambda_1 \lambda_2)$ which could be expressed as:

$$R_{\lambda_1 \lambda_2}^{SS_z}(x_i, k_\perp) = \sum_{s_1, s_2} \langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_\perp, m_1) | s_1 \rangle \langle \lambda_2 | \mathcal{R}_M^\dagger(x_2, -k_\perp, m_2) | s_2 \rangle \langle \frac{1}{2} s_1 \frac{1}{2} s_2 | SS_z \rangle, \tag{7}$$

where $|s_i\rangle$ are the Pauli spinors, and \mathcal{R}_M is the Melosh transformation operator:

$$\mathcal{R}_M(x, k_\perp, m_i) = \frac{m_i + x_i M_0 + i \vec{\sigma} \cdot \vec{k}_\perp \times \vec{n}}{\sqrt{(m_i + x_i M_0)^2 + k_\perp^2}}, \tag{8}$$

with

$$M_0^2 = \frac{m_1^2 + k_\perp^2}{x_1} + \frac{m_2^2 + k_\perp^2}{x_2}, \tag{9}$$

and $\vec{n} = (0, 0, 1)$. Actually, if we set $x_1 = 1 - x$ and $x_2 = x$, Eq. (7) can be given by a covariant form [3].

$$R_{\lambda_1 \lambda_2}^{SS_z}(x, k_\perp) = \frac{\sqrt{k_1^+ k_2^+}}{\sqrt{2} \widetilde{M}_0} \bar{u}(k_1, \lambda_1) \Gamma v(k_2, \lambda_2), \tag{10}$$

with

$$\begin{aligned}
\widetilde{M}_0 &\equiv \sqrt{M_0^2 - (m_1 - m_2)^2}, \\
\sum_\lambda u(k, \lambda) \bar{u}(k, \lambda) &= \frac{m + \not{k}}{k^+}, \quad \sum_\lambda v(k, \lambda) \bar{v}(k, \lambda) = \frac{m - \not{k}}{k^+},
\end{aligned} \tag{11}$$

where Γ stands for

$$\begin{aligned}\Gamma &= \gamma_5 \quad (\text{pseudoscalar}, S = 0), \\ \Gamma &= -\not{\epsilon}(S_z) + \frac{\hat{\epsilon} \cdot (p_1 - p_2)}{M_0 + m_1 + m_2} \quad (\text{vector}, S = 1),\end{aligned}\tag{12}$$

and

$$\begin{aligned}\hat{\epsilon}^\mu(\pm 1) &= \left[\frac{2}{P^+} \vec{\epsilon}_\perp(\pm 1) \cdot \vec{P}_\perp, 0, \vec{\epsilon}_\perp(\pm 1) \right], \quad \vec{\epsilon}_\perp(\pm 1) = \mp(1, \pm i)/\sqrt{2}, \\ \hat{\epsilon}^\mu(0) &= \frac{1}{M_0} \left(\frac{-M_0^2 + P_\perp^2}{P^+}, P^+, P_\perp \right).\end{aligned}\tag{13}$$

The normalization condition of the meson state is given by

$$\langle M(P', S', S'_z) | M(P, S, S_z) \rangle = 2(2\pi)^3 P^+ \delta^3(\vec{P}' - \vec{P}) \delta_{S'S} \delta_{S'_z S_z},\tag{14}$$

which leads to

$$\int \frac{dx d^2 k_\perp}{2(2\pi)^3} |\phi(x, k_\perp)|^2 = 1.\tag{15}$$

In principle, the momentum distribution amplitude $\phi(x, k_\perp)$ can be obtained by solving the light-front QCD bound state equation [17, 18]. However, before such first-principle solutions are available, we would have to be contented with phenomenological amplitudes. One example that has been often used in the literature is the Gaussian type wave function:

$$\phi(x, k_\perp) = N \sqrt{\frac{dk_z}{dx}} \exp\left(-\frac{\vec{k}^2}{2\omega_M^2}\right),\tag{16}$$

where $N = 4 \left(\frac{\pi}{\omega_M^2}\right)^{\frac{3}{4}}$, $\vec{k} = (k_\perp, k_z)$, k_z is defined through

$$1 - x = \frac{e_1 - k_z}{e_1 + e_2} \quad x = \frac{e_2 + k_z}{e_1 + e_2}, \quad e_i = \sqrt{m_i^2 + \vec{k}^2}\tag{17}$$

by

$$k_z = \left(x - \frac{1}{2}\right) M_0 + \frac{m_1^2 - m_2^2}{2M_0}.\tag{18}$$

and

$$\frac{dk_z}{dx} = \frac{e_1 e_2}{x(1-x)M_0}\tag{19}$$

is the Jacobian of the transformation from (x, k_\perp) to \vec{k} . In particular, with appropriate parameters, this wave function in Eq. (16) describes satisfactorily the pion elastic form factor up to $p^2 \sim 10 \text{ GeV}^2$ [19].

2.2 Decay constants

In order to use the light front bound states in Eq. (3) to calculate the matrix elements in Eq. (2), we need to fix the parameter ω in the wave function of Eq. (16). This parameter can be determined by the corresponding pseudoscalar and vector mesonic decay constants, defined by $\langle 0|A^\mu|P\rangle = if_P P^\mu$ and $\langle 0|V^\mu|V\rangle = f_V M_V \epsilon^\mu$, respectively.

From Eq. (3), one has

$$\begin{aligned} \langle 0|\bar{q}_2\gamma^+\gamma_5 q_1|P\rangle &= \int [d^3 p_1][d^3 p_2] 2(2\pi)^3 \delta^3(P - p_1 - p_2) \phi_P(x, k_\perp) R_{\lambda_1 \lambda_2}^{00}(x, k_\perp) \\ &\times \langle 0|\bar{q}_2\gamma^+\gamma_5 q_1|q_1(p_1, \lambda_1)\bar{q}_2(p_2, \lambda_2)\rangle, \end{aligned} \quad (20)$$

It is straightforward to show that

$$f_P = 4 \frac{\sqrt{3}}{\sqrt{2}} \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \phi_P(x, k_\perp) \frac{\mathcal{A}}{\sqrt{\mathcal{A}^2 + k_\perp^2}}, \quad (21)$$

where

$$\mathcal{A} = m_1 x + m_2(1 - x). \quad (22)$$

Note that the factor $\sqrt{3}$ in (21) arises from the color factor implicitly in the meson wave function.

Similarly, the vector-meson decay constant is found to be

$$\begin{aligned} f_V &= 4 \frac{\sqrt{3}}{\sqrt{2}} \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \frac{\phi_V(x, k_\perp)}{\sqrt{\mathcal{A}^2 + k_\perp^2}} \frac{1}{M_0} \left\{ x(1 - x)M_0^2 + m_1 m_2 + k_\perp^2 \right. \\ &\quad \left. + \frac{\mathcal{B}}{2W_V} \left[\frac{m_1^2 + k_\perp^2}{1 - x} - \frac{m_2^2 + k_\perp^2}{x} - (1 - 2x)M_0^2 \right] \right\}, \end{aligned} \quad (23)$$

where

$$\mathcal{B} = x m_1 - (1 - x) m_2, \quad W_V = M_0 + m_1 + m_2. \quad (24)$$

If the decay constant is experimentally known, one can determine the parameter ω in the light-front wave function.

2.3 Tensor form factor for $P \rightarrow P$ transition

The hadronic matrix element of the tensor operator for the $P \rightarrow P$ transition could be newly parametrized in terms of the initial meson momentum $p + q$ and final meson momentum q , that is,

$$\langle P_2(q)|\bar{q}i\sigma^{\mu\nu}p_\nu Q|P_1(p+q)\rangle = \frac{F_T}{M_{P_1} + M_{P_2}} [(p + 2q)^\mu p^2 - (M_{P_1}^2 - M_{P_2}^2)p^\mu], \quad (25)$$

where p is the momentum transfer. For $P_1 = (Q_1 \bar{q})$ and $P_2 = (Q_2 \bar{q})$, the relevant quark momentum variables are

$$\begin{aligned} p_{Q_1}^+ &= (1-x)(p+q)^+, \quad p_{\bar{q}}^+ = x(p+q)^+, \quad p_{Q_{1\perp}} = (1-x)q_\perp + k_\perp, \quad p_{\bar{q}\perp} = xq_\perp - k_\perp, \\ p_{Q_2}^+ &= (1-x')q^+, \quad p_{\bar{q}}'^+ = x'q^+, \quad p_{Q_{2\perp}} = (1-x')q_\perp + k'_\perp, \quad p_{\bar{q}\perp}' = x'q_\perp - k'_\perp, \end{aligned} \quad (26)$$

where x (x') is the momentum fraction of the anti-quark in the initial (final) state. At the quark loop, this anti-quark is the spectator, thus it requires that

$$p_{\bar{q}}'^+ = p_{\bar{q}}^+, \quad p_{\bar{q}\perp}' = p_{\bar{q}\perp}. \quad (27)$$

To calculate the matrix element in Eq. (25), we take a Lorentz frame where the transverse momentum $p_\perp = 0$ such that $p^2 = p^+p^- \geq 0$ covers the entire physical region of the momentum transfer [20].

Consider the “good” component of $\mu = +$, we have

$$\begin{aligned} \langle P_2 | \bar{q} i \sigma^{+\nu} p_\nu Q | P_1 \rangle &= \sqrt{\frac{x}{x'}} \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \phi_{P_2}^*(x', k_\perp) \phi_{P_1}(x, k_\perp) \frac{1}{2\widetilde{M}_{0P_2} \widetilde{M}_{0P_1} \sqrt{(1-x')(1-x)}} \\ &\times \text{Tr} \left[\gamma_5 (\not{p}_2 + m_{Q_2}) \sigma^{+\nu} p_\nu (\not{p}_1 + m_{Q_1}) \gamma_5 (\not{p}_{\bar{q}} - m_{\bar{q}}) \right]. \end{aligned} \quad (28)$$

The trace in the above expression can be easily carried out. By using Eqs. (25) and (28), the form factor F_T is found to be

$$F_T(p^2) = \frac{M_{P_1} + M_{P_2}}{(1+2r)p^2 - (M_{P_1}^2 - M_{P_2}^2)} \int dx' d^2 k_\perp \frac{\phi_{P_2}^*(x', k_\perp) \phi_{P_1}(x, k_\perp)}{\widetilde{M}_{0P_2} \widetilde{M}_{0P_1} \sqrt{(1-x')(1-x)}} A \quad (29)$$

where $x = x'r/(1+r)$ and

$$\begin{aligned} A &= \frac{1}{xx'(1-x)(1-x')} \left\{ (xm_{Q_1} + (1-x)m_{\bar{q}})(x'm_{Q_2} + (1-x')m_{\bar{q}}) \right. \\ &\quad [x(1-x')m_{Q_1} - x'(1-x)m_{\bar{q}}] + k_\perp^2 \left[x'(1-x')(2x-1)m_{Q_2} \right. \\ &\quad \left. \left. + (x-x')(x+x'-2xx')m_{\bar{q}} + x(1-x)(1-2'x)m_{Q_1} \right] \right\}. \end{aligned} \quad (30)$$

At the maximum recoil of $p^2 = 0$, we have $x = x'$ and get

$$\begin{aligned} F_T(0) &= -\frac{1}{M_{P_1} - M_{P_2}} \int dx d^2 k_\perp \phi_{P_2}^*(x, k_\perp) \phi_{P_1}(x, k_\perp) \\ &\times \frac{1}{x^2(1-x)\widetilde{M}_{0P_2}\widetilde{M}_{0P_1}} \left\{ (xm_{Q_1} + (1-x)m_{\bar{q}})(xm_{Q_2} + (1-x)m_{\bar{q}}) \right. \\ &\quad \left. [x(1-x)m_{Q_1} - x(1-x)m_{Q_2}] + k_\perp^2 x(1-x)(1-2x)(m_{Q_1} - m_{Q_2}) \right\}. \end{aligned} \quad (31)$$

2.4 Tensor form factors for $P \rightarrow V$ transition

Similarly, the tensor form factors $F_{1,2,3}$ for the $P \rightarrow V$ transition are defined as:

$$\begin{aligned}\langle V(q, \epsilon) | \bar{q} i \sigma^{\mu\nu} p_\nu Q | P(p+q) \rangle &= -i \varepsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* q_\alpha p_\beta F_1, \\ \langle V(q, \epsilon) | \bar{q} i \sigma^{\mu\nu} p_\nu \gamma_5 Q | P(p+q) \rangle &= \left[(m_P^2 - m_V^2) \epsilon^{*\mu} - (\epsilon^* \cdot p)(p+2q)^\mu \right] F_2 \\ &\quad + \epsilon^* \cdot p \left[p^\mu - \frac{p^2}{m_P^2 - m_V^2} (p+2q)^\mu \right] F_3, \end{aligned} \quad (32)$$

where q and $p+q$ are the four momenta of the vector and pseudoscalar mesons, and ϵ^μ is the meson polarization vector, given by

$$\varepsilon^\mu(\pm 1) = \left[\frac{2 \vec{\varepsilon}_\perp \cdot \vec{q}_\perp}{q^+}, 0, \vec{\varepsilon}_\perp \right], \quad \varepsilon^\mu(0) = \frac{1}{M_V} \left(\frac{-M_V^2 + q_\perp^2}{q^+}, q^+, q_\perp \right), \quad (33)$$

respectively. The calculation of the $P \rightarrow V$ form factors is more subtle than that in the $P \rightarrow P$ case. Here we use the two-component form of the light-front quark field [21]. For the “good” component of $\mu = +$, the tensor current in Eq. (1) can be written as:

$$\bar{q} i \sigma^{+\nu} p_\nu (1 + \gamma_5) Q = \{ q_+^\dagger \gamma^0 (1 + \gamma_5) Q_- - q_-^\dagger \gamma^0 (1 + \gamma_5) Q_+ \} p^+ \quad (34)$$

where the $q_+(Q_+)$ and $q_-(Q_-)$ are the light-front up and down components of the quark fields, expressed by [21]

$$q_+ = \begin{pmatrix} \chi_q \\ 0 \end{pmatrix} \quad (35)$$

and

$$q_- = \frac{1}{i \partial^+} (i \alpha_\perp \cdot \partial_\perp + \beta m_q) q_+ = \begin{pmatrix} 0 \\ \frac{1}{\partial^+} (\tilde{\sigma}_\perp \cdot \partial_\perp + m_q) \chi_q \end{pmatrix}, \quad (36)$$

respectively. Then, the tensor operator in the Eq. (34) could be written as follow:

$$\begin{aligned} q_+^\dagger \gamma^0 (1 + \gamma_5) Q_- &= -i \chi_q^\dagger (1 - \sigma_3) \frac{1}{\partial^+} (\tilde{\sigma}_\perp \cdot \vec{\partial}_\perp + m_q) \chi_Q, \\ q_-^\dagger \gamma^0 (1 + \gamma_5) Q_+ &= i \left[\frac{1}{\partial^+} \chi_q^\dagger (\tilde{\sigma}_\perp \cdot \overleftarrow{\partial}_\perp + m_q) \right] (1 + \sigma_3) \chi_Q, \end{aligned} \quad (37)$$

where $\chi_{q(Q)}$ is a two-component spinor field and σ is the Pauli matrix.

Let $P = (Q_1 \bar{q})$ and $V = (Q_2 \bar{q})$. By the use of Eqs. (3), (34) and (37), Eq. (32) can be reduced to

$$\begin{aligned} \langle V(q, \epsilon) | \bar{q} i \sigma^{+\nu} p_\nu Q | P(p+q) \rangle \\ = -i \int [dk_1] [dk'_1] [dk_2] 2(2\pi)^3 \delta^3(p+q-k_1-k_2) 2(2\pi)^3 \delta^3(q-k'_1-k_2) \end{aligned}$$

$$\begin{aligned}
& \times \sum \Phi_P^{\lambda_1 \lambda_2}(k_1, k_2, \lambda_1, \lambda_2) \sum \Phi_V^{*\lambda'_1 \lambda_2}(k'_1, k_2, \lambda'_1, \lambda_2) \\
& \times \chi_{\lambda'_1}^+ \left\{ \frac{1}{k_1^+} (\tilde{\sigma}_\perp \cdot k_{1\perp} + im_Q) + \frac{1}{k_1^{+\prime}} (\tilde{\sigma}_\perp \cdot k'_{1\perp} + im_q) \right\} \chi_{-\lambda'_2}, \\
& \langle V(q) | \bar{q} i \sigma^{\mu\nu} p_\nu \gamma_5 Q | P(p+q) \rangle \\
& = -i \int [dk_1] [dk'_1] [dk_2] 2(2\pi)^3 \delta^3(p+q-k_1-k_2) 2(2\pi)^3 \delta^3(q-k'_1-k_2) \\
& \times \sum \Phi_P^{\lambda_1 \lambda_2}(k_1, k_2, \lambda_1, \lambda_2) \sum \Phi_V^{*\lambda'_1 \lambda_2}(k'_1, k_2, \lambda'_1, \lambda_2) \\
& \times \chi_{\lambda'_1}^+ \left\{ \sigma_3 \frac{-1}{k_1^+} (\tilde{\sigma}_\perp \cdot k_{1\perp} + im_Q) + \frac{1}{k_1^{+\prime}} (\tilde{\sigma}_\perp \cdot k'_{1\perp} + im_q) \sigma_3 \right\} \chi_{-\lambda'_2}. \quad (38)
\end{aligned}$$

The form factor F_1 is only related to the 1^- intermediate state with the vector daughter meson but F_2 and F_3 mix with 1^+ and 0^+ states. Therefore, we have to calculate the matrix elements for 1^+ and 0^+ states with different vector meson polarizations. For $\varepsilon^\mu(\pm 1)$ and $\varepsilon^\mu(0)$, $R_{\lambda_1 \lambda_2}^{SS_z}(x, k_\perp)$ in $\Phi_V^{*\lambda'_1 \lambda_2}(k'_1, k_2, \lambda'_1, \lambda_2)$ can be written respectively as:

$$\begin{aligned}
R_{\lambda_1 \lambda_2}^{SS_z}(x, k_\perp) &= \frac{\sqrt{p_1^+ p_2^+}}{\sqrt{2} \widetilde{M}_{0V}} \chi_{\lambda_1}^+ \left\{ \frac{(1-x) \sigma_\perp \cdot P_\perp + \sigma_\perp \cdot k_\perp - im_{Q_2}}{(1-x) P^+} \sigma_\perp \cdot \epsilon_\perp \right. \\
&\quad - 2 \frac{\epsilon_\perp \cdot P_\perp}{P^+} + \sigma_\perp \cdot \epsilon_\perp \frac{x \sigma_\perp \cdot P_\perp - \sigma_\perp \cdot k_\perp - im_{\bar{q}}}{x P^+} \\
&\quad \left. + 2 \frac{\epsilon_\perp \cdot k_\perp}{W_V x (1-x) P^+} [-i \sigma_\perp \cdot k_\perp - x m_{Q_2} + (1-x) m_2] \right\} \chi_{-\lambda_2} \quad (39)
\end{aligned}$$

and

$$\begin{aligned}
R_{\lambda_1 \lambda_2}^{SS_z}(x, k_\perp) &= \frac{\sqrt{p_1^+ p_2^+}}{\sqrt{2} \widetilde{M}_{0V}} \frac{1}{M_{0V} P^+} \chi_{\lambda_1}^+ \left\{ M_{0V}^2 - P_\perp^2 \right. \\
&\quad + \frac{1}{x(1-x)} [x(1-x)(\sigma_\perp \cdot P_\perp)(\sigma_\perp \cdot P_\perp) \\
&\quad + (\sigma_\perp \cdot k_\perp)(\sigma_\perp \cdot k_\perp) - i(m_{Q_2} - m_{\bar{q}})(\sigma_\perp \cdot k_\perp) + m_{Q_2} m_{\bar{q}}] \\
&\quad + \frac{1}{2W_V x(1-x)} \left[\frac{m_{Q_2}^2 + k_\perp^2}{1-x} - \frac{m_{\bar{q}}^2 + k_\perp^2}{x} - (1-2x) M_{0V}^2 \right] \\
&\quad \left. \times [i(\sigma_\perp \cdot k_\perp) + x m_{Q_2} - (1-x) m_{\bar{q}}] \right\} \chi_{-\lambda_2} \quad (40)
\end{aligned}$$

where,

$$W_V = M_{0V} + m_{Q_2} + m_{\bar{q}}, \quad M_{0V}^2 = \frac{m_{Q_2}^2 + k_\perp^2}{1-x'} + \frac{m_{\bar{q}}^2 + k_\perp^2}{x'}. \quad (41)$$

Since the form factor F_1 is only related to $\varepsilon^\mu(-1)$, we can find the matrix element as follow:

$$\langle V(q, \epsilon) | \bar{q} i \sigma^{\mu\nu} p_\nu Q | P(p+q) \rangle = F_1 \epsilon^{ij} \epsilon_i^* q_j. \quad (42)$$

Using Eqs. (38) and (42), we obtain

$$F_1 = \int dx' d^2 k_\perp N_V \phi_V^*(x', k_\perp) N_P \phi_P(x, k_\perp) (A_1 + A_2), \quad (43)$$

where

$$N\phi(x, k_\perp) = 4 \left(\frac{\pi}{\omega_M^2} \right)^{\frac{3}{4}} \frac{1}{\sqrt{2} \sqrt{M_0^2 - (m_Q - m_{\bar{q}})^2}} \sqrt{\frac{e_Q e_{\bar{q}}}{M_0}} \exp \left(-\frac{\vec{k}_\perp^2}{2\omega_M^2} \right), \quad (44)$$

$$\begin{aligned} A_1 &= \frac{2}{xx'^2(1-x')^2(1-x)^2} \left\{ (1-2x'+xx')(x'+x-2x'x)k_\perp^2 \right. \\ &\quad + [x'm_{Q_2} + (1-x')m_{\bar{q}}][2xx'(1-x)(1-x')m_{Q_1} \\ &\quad + (1-x)(1-x')(x+x'-2xx')m_{\bar{q}} + x'(1-x)(x-x')m_{Q_2}] \\ &\quad \left. + \frac{k_\perp^2}{W_V} [2xx'(1-x)(1-x')m_{Q_1} + 2xx'(1-x)(1-x')m_{Q_2}] \right\}, \\ A_2 &= \frac{2(x-x')\Theta_P k_\perp^2}{xx'^2(1-x')^2(1-x)^2} \left\{ k_\perp^2(x'+x-2x'x) \right. \\ &\quad + [x(1-x')m_{Q_1} - x'(1-x)m_{Q_2}][xm_{Q_1} - x'm_{Q_2} + (x'-x)m_{\bar{q}}] \\ &\quad \left. + (x+x'-2xx')[x'm_{Q_2} + (1-x')m_{\bar{q}}][xm_{Q_1} + (1-x)m_{\bar{q}}] \right\}, \end{aligned} \quad (45)$$

and

$$\Theta_P = \frac{1}{\phi_P(x, k_\perp^2)} \frac{d\phi_P(x, k_\perp^2)}{dk_\perp^2}. \quad (46)$$

At $p^2 = 0$ in which we have $x = x'$, it can be shown from Eq. (43) that

$$\begin{aligned} F_1(0) &= \int dx d^2 k_\perp N_V \phi_V(x, k_\perp) N_P \phi_P^*(x, k_\perp) \frac{4}{x^2(1-x)^2} \left\{ (1-x)k_\perp^2 \right. \\ &\quad \left. + [xm_{Q_2} + (1-x)m_{\bar{q}}][xm_{Q_1} + (1-x)m_{\bar{q}}] + \frac{k_\perp^2}{W_V} x(m_{Q_1} + m_{Q_2}) \right\}. \end{aligned} \quad (47)$$

We note that Eq. (47) is the same as that in Ref. [6].

The calculations of the form factors F_2 and F_3 are more complex than that of F_1 because they cannot be determined separately. From Eq. (32), we have that

$$\begin{aligned} &\langle V(q, \epsilon) | \bar{q} i \sigma^{+\nu} p_\nu \gamma_5 Q | P(p+q) \rangle \\ &= \left\{ -(1+2r)F_2 + \left[1 - \frac{p^2}{M_P^2 - M_V^2} (1+2r) \right] F_3 \right\} \frac{1}{r} (\epsilon_\perp \cdot q_\perp), \end{aligned} \quad (48)$$

and

$$\begin{aligned} &\langle V(q, \epsilon) | \bar{q} i \sigma^{+\nu} p_\nu \gamma_5 Q | P(p+q) \rangle \\ &= \left[(m_P^2 - m_V^2) \frac{r}{M_V} - \frac{1+2r}{2M_V} \left(\frac{rM_P^2}{1+r} - M_V^2 - \frac{M_V^2}{r} \right) \right] F_2 \\ &\quad + \frac{1}{2M_V} \left(\frac{rM_P^2}{1+r} - M_V^2 - \frac{M_V^2}{r} \right) \times \left[1 - \frac{p^2}{M_P^2 - M_V^2} (1+2r) \right] F_3, \end{aligned} \quad (49)$$

for the transverse and longitudinal vector mesons polarizations of $\varepsilon^\mu(1)$ and $\varepsilon^\mu(0)$, respectively, where

$$r = \frac{q^+}{p^+} = \frac{-(p^2 + M_V^2 - M_P^2) + \sqrt{(p^2 + M_V^2 - M_P^2)^2 - 4p^2 M_V^2}}{2p^2}. \quad (50)$$

Using Eqs. (38), (39) and (40), we get

$$\begin{aligned} & \left\{ -(1+2r)F_2 + \left[1 - \frac{p^2}{M_P^2 - M_V^2}(1+2r) \right] F_3 \right\} \frac{1}{r} \\ &= \int dx' d^2 k_\perp N_V \phi_V^*(x', k_\perp) N_P \phi_P(x, k_\perp) (B_1 + B_2) \equiv h(p^2) \end{aligned} \quad (51)$$

and

$$\begin{aligned} & \left[(m_P^2 - m_V^2) \frac{r}{M_V} - \frac{1+2r}{2M_V} \left(\frac{rM_B^2}{1+r} - M_V^2 - \frac{M_V^2}{r} \right) \right] F_2 \\ &+ \frac{1}{2M_V} \left(\frac{rM_B^2}{1+r} - M_V^2 - \frac{M_V^2}{r} \right) \left[1 - \frac{p^2}{M_P^2 - M_V^2}(1+2r) \right] F_3 \\ &= \int dx' d^2 k_\perp N_V \phi_V^*(x', k_\perp) N_P \phi_P(x, k_\perp) C \equiv g(p^2) \end{aligned} \quad (52)$$

where

$$\begin{aligned} B_1 &= \frac{2}{xx'^2(1-x')^2(1-x)^2} \left\{ (2x' - 2x'^2 - x + xx')(x' + x - 2xx')k_\perp^2 \right. \\ &\quad - \frac{1}{W_V} \left[2xx'(1-x)(1-x')m_{Q_1} + 2(1-x')(x' - x)(x' + x - 2xx')m_{\bar{q}} \right. \\ &\quad + 2x'(1-x')(x - x')^2 m_{Q_2} \left. \right] k_\perp^2 - \left([x'm_{Q_2} + (1-x')m_{\bar{q}}][2xx'(1-x)(1-x')m_{Q_1} \right. \\ &\quad + (1-x)(1-x')(x + x' - 2xx')m_{\bar{q}} - x'(1-x)(x - x')m_{Q_2}] \left. \right) \left. \right\}, \\ B_2 &= \frac{2(x - x')\Theta_P k_\perp^2}{xx'^2(1-x')^2(1-x)^2} \left\{ (2x' - 1)(x' + x - 2xx')k_\perp^2 \right. \\ &\quad - \frac{2}{W_V} \left[x(1-x)(2x' - 1)m_{Q_1} + (x' - x)(x' + x - 2xx')m_{\bar{q}} \right. \\ &\quad + x'(1-x')(2x - 1)m_{Q_2} \left. \right] k_\perp^2 + \left[[x(1-x')m_{Q_1} - x'(1-x)m_{Q_2}] \right. \\ &\quad \times [(2x' - 1)m_{Q_1} + x'm_{Q_2} + (x + x' - 2xx')m_{\bar{q}}] \\ &\quad - (x + x' - 2xx')[x'm_{Q_2} + (1-x')m_{\bar{q}}][xm_{Q_1} + (1-x)m_{\bar{q}}] \\ &\quad - \frac{2}{W_V} \left[x^2 x'(1-x')m_q m_b^2 - x^2(1-x')^2 m_{\bar{q}} m_{Q_1}^2 \right. \\ &\quad - x(1-x)(1-x')^2 m_{Q_1} m_{\bar{q}}^2 + xx'^2(1-x)m_{Q_1} m_{Q_2}^2 \\ &\quad + x'^2(1-x)^2 m_{\bar{q}} m_{Q_2}^2 - x'(1-x)^2(1-x')m_{Q_2} m_{\bar{q}}^2 \left. \right] \left. \right\}, \\ C &= \frac{2}{xx'^2(1-x')^2(1-x)^2 M_{0V}} \left\{ \left[x'(1-x')M_{0V}^2 + k_\perp^2 + m_{Q_2} m_{\bar{q}} \right] k_\perp^2 (x' + x - 2xx') \right. \end{aligned}$$

$$\begin{aligned}
& + \left[x'(1-x')M_{0V}^2 + k_\perp^2 + m_{Q_2}m_{\bar{q}} \right] \\
& \times (xm_{Q_1} + (1-x)m_{\bar{q}})(x'(1-x)m_{Q_2} + x(1-x')m_{Q_1}) \\
& + \frac{1}{2W_V} \left[\frac{m_{Q_2}^2 + k_\perp^2}{1-x'} - \frac{m_{\bar{q}}^2 + k_\perp^2}{x'} - (1-2x')M_{0V}^2 \right] \\
& \times (xm_{Q_1} + (1-x)m_{\bar{q}})(x'm_{Q_2} - (1-x')m_{\bar{q}})(x'(1-x)m_{Q_2} + x(1-x')m_{Q_1}) \\
& - (m_{Q_2} - m_{\bar{q}}) \\
& \times \left(x(1-x)(2x' - 1)m_{Q_1} + (1-x)(x' + x - 2xx')m_{\bar{q}} - x'(1-x)m_{Q_2} \right) k_\perp^2 \\
& + \frac{1}{2W_V} \left[\frac{m_{Q_2}^2 + k_\perp^2}{1-x'} - \frac{m_{\bar{q}}^2 + k_\perp^2}{x'} - (1-2x')M_{0V}^2 \right] \\
& \times \left(x'(1-x')(2x - 1)m_{Q_2} + (x' - x)(x' + x - 2xx')m_{\bar{q}} \right. \\
& \left. + x(1-x)(2x' - 1)m_{Q_1} \right) k_\perp^2 \Big\}. \tag{53}
\end{aligned}$$

The physical kinematic range of $p^2 : 0 \rightarrow (M_P - M_V)^2$ corresponds to

$$\frac{x}{x'} : 1 \rightarrow \frac{M_V}{M_P}. \tag{54}$$

From Eqs. (51) and (52), it is not difficult to find F_2 and F_3 as follows:

$$\begin{aligned}
(M_P^2 - M_V^2)F_2 &= \frac{M_V}{r}h(p^2) - \frac{g(p^2)}{2} \left(\frac{r}{1+r}M_P^2 - \frac{1+r}{r}M_V^2 \right), \\
\left[1 - \frac{p^2}{M_P^2 - M_V^2} \right] F_3 &= \frac{M_V}{M_P^2 - M_V^2} \frac{1+2r}{r} h(p^2) \\
&+ g(p^2) \left[r - \frac{1}{M_P^2 - M_V^2} \frac{1+2r}{2} \left(\frac{r}{1+r}M_P^2 - \frac{1+r}{r}M_V^2 \right) \right]. \tag{55}
\end{aligned}$$

At the maximum recoil of $p^2 = 0$, $F_2(0)$ and $F_3(0)$ are simply given by

$$\begin{aligned}
F_2(0) &= - \int dx d^2k_\perp N_V \phi_V^*(x, k_\perp) N_P \phi_P(x, k_\perp) \\
&\quad \frac{4}{x^2(1-x)^2} \left\{ xk_\perp^2 - \frac{k_\perp^2}{W_V} xm_{Q_1} - [xm_{Q_2} + (1-x)m_{\bar{q}}][xm_{Q_1} + (1-x)m_{\bar{q}}] \right\}, \\
F_3(0) &= \int dx d^2k_\perp N_V \phi_V^*(x, k_\perp) N_P \phi_P(x, k_\perp) \\
&\quad \frac{M_V}{M_P^2 - M_V^2} \frac{2}{x^2(1-x)^3 M_{0V}} \left\{ \left[x(1-x)M_{0V}^2 + k_\perp^2 + m_{Q_2}m_{\bar{q}} \right] \right. \\
&\quad \times [(xm_{Q_1} + (1-x)m_{\bar{q}})(m_{Q_2} + m_{Q_1}) + 2k_\perp^2] \\
&\quad + \frac{m_{Q_2} + m_{Q_1}}{2W_V} \left[\frac{m_{Q_2}^2 + k_\perp^2}{1-x} - \frac{m_{\bar{q}}^2 + k_\perp^2}{x} - (1-2x)M_{0V}^2 \right] \\
&\quad \times [(xm_{Q_1} + (1-x)m_{\bar{q}})(xm_{Q_2} - (1-x)m_{\bar{q}}) + (2x-1)k_\perp^2] \\
&\quad \left. - (m_{Q_2} - m_{\bar{q}}) \left((2x-1)m_{Q_1} + 2(1-x)m_{\bar{q}} - m_{Q_2} \right) k_\perp^2 \right\} + \frac{F_2(0)}{2}. \tag{56}
\end{aligned}$$

Thus, we have evaluated all the tensor form factors of F_T , F_1 , F_2 and F_3 in the whole physical region of the momentum transfer with the light front quark model.

Table 1: Parameters ω_M ($M = \pi, \rho, K, K^*, B$ and B^*).

wave function	$m_{u,d}$	ω_π	ω_ρ	m_s	ω_K	ω_{K^*}	m_b	ω_B	ω_{B^*}
Gaussian	0.25	0.33	0.31	0.40	0.38	0.31	4.8	0.55	0.55

3 Numerical Results

We now calculate numerically the tensor form factors in the LFQM. The parameters in the light front wave functions are fixed by other hadronic properties. We use the decay constants in Eqs. (21) and (23) to determine the parameter ω . The measured decay constants of light pseudoscalar and vector mesons are

$$f_\pi = 132 \text{ MeV}, \quad f_\rho = 216 \text{ MeV}, \quad f_K = 160 \text{ MeV}, \quad f_{K^*} = 210 \text{ MeV}. \quad (57)$$

Since the decay constants of heavy mesons are unknown experimentally, we have to rely on various calculations in QCD-motivated models [22, 23, 24]. We shall take

$$f_B = 185 \text{ MeV}, \quad f_{B^*} = 205 \text{ MeV}. \quad (58)$$

In Table 1, we list the parameters ω_M ($M = \pi, \rho, K, K^*, B$ and B^*) fitted by the decay constants in Eqs. (59) and (60) with the Gaussian-type wave functions. Note that the quark masses given in Table 1 are chosen to the commonly used values in relativistic quark models.

We now show the form factors F_T, F_1, F_2 and F_3 in the entire physical region $0 \leq p^2 \leq (M_i - M_f)^2$. The p^2 dependences of F_T for $B \rightarrow K$ and $B \rightarrow \pi$ are shown in Figures 1 and 2, respectively. From Figure 1, one can see that F_T will decrease as p^2 increases near the zero recoil point. This is reasonable because the matrix elements depend on the overlap integral of the initial and final meson wave functions. For the heavy to light transition, the internal momentum distribution of the heavy meson $\phi(x, k_\perp)$ has a narrow peak near $x = 0$, whereas the peak of $\phi(x, k_\perp)$ for the light meson has a larger width than that for the heavy one. Therefore, the maximum overlapping of these two wave functions occurs away from the zero recoil point. At the maximum recoil of $p^2 = 0$, we have the values

$$F_T^{B\pi}(0) = 0.27, \quad F_T^{BK}(0) = 0.36. \quad (59)$$

We note that the values in Eq. (61) are close to those in the light cone QCD sum rule models [25, 26, 27].

Table 2: The values of Λ_1 and Λ_2 for the corresponding $F_i(0)$ in $B \rightarrow M$ ($M = \pi, K, \rho, K^*$).

M	π	K	ρ			K^*		
$F_i(p^2)$	F_T	F_T	F_1	F_2	F_3	F_1	F_2	F_3
$p^2 = 0$	0.27	0.36	0.54	0.27	0.18	0.63	0.32	0.21
$\Lambda_1(\text{GeV})$	4.32	4.36	4.03	6.04	4.45	3.96	6.17	4.78
$\Lambda_2(\text{GeV})$	5.38	5.29	5.43	12.45	5.84	5.18	11.02	6.17

The form factors $F_{1,2,3}$ as a function of p^2 for $B \rightarrow K^*$ and $B \rightarrow \rho$ transitions are depicted in Figures 3 and 4, respectively. At $p^2 = 0$, we have

$$\begin{aligned}
B \rightarrow K^* : \quad & F_1^{BK^*}(0) = 0.63, \quad F_2^{BK^*}(0) = 0.32, \quad F_3^{BK^*}(0) = 0.21 \\
B \rightarrow \rho : \quad & F_1^{B\rho}(0) = 0.54, \quad F_2^{B\rho}(0) = 0.27, \quad F_3^{B\rho}(0) = 0.19.
\end{aligned} \tag{60}$$

To compare our results with those in the literature, we may fit approximately the p^2 behaviors of the form factors in Figures 1-4 as pole-like forms:

$$F_i(p^2) = \frac{F_i(0)}{1 - p^2/\Lambda_1^2 + p^4/\Lambda_2^4} \quad (i = T, 1, 2, 3) \tag{61}$$

where the values of Λ_1 and Λ_2 are listed in the Table 2 with F_T for $B \rightarrow \pi, K$ and $F_{1,2,3}$ for $B \rightarrow \rho, K^*$.

As shown in Table 3, we see that our results of $F_i(0)$ agree well with those in the literature [6, 28, 29]. We remark that the relations among the form factors for the $P \rightarrow V$ transition in Ref. [30] can be tested in our calculations. We find that the relation of Eqs. (5)-(7) in Ref. [30] are well satisfied at $p^2 \rightarrow 0$ and hold with 20% accuracy at $p^2 \leq 10 \text{ GeV}^2$. However, they are violated for p^2 larger than 10 GeV^2 . Finally, we note that the relation $F_1(0) = 2 F_2(0)$ [28] is satisfied and $F_1(p^2)$ increases as p^2 faster than $2 F_2(p^2)$.

4 Conclusions

The tensor form factors of F_T and $F_{1,2,3}$ for $P \rightarrow P$ and $P \rightarrow V$ transitions have been studied in the LFQM. These form factors are evaluated in the entire physical momentum transfer region of $0 \leq p^2 \leq (M_i - M_f)^2$. We have used the values of the decay constants and the constituent quark masses to fix the parameters ω_M appearing in the wave functions. Thus, there are no more degree of freedom to adjust the light front wave functions.

Table 3: Comparison of different works for the tensor form factors at $p^2 = 0$

$B \rightarrow \pi(\rho)$	This work	[6]	[28]	[29]
$F_T(0)$	0.27	-	-	-
$F_1(0)$	0.54	0.56	0.58 ± 0.08	-
$F_2(0)$	0.27	-	0.29 ± 0.04	-
$F_3(0)$	0.19	-	0.20 ± 0.03	-
$B \rightarrow K(K^*)$	This work	[6]	[28]	[29]
$F_T(0)$	0.36	-	-	-
$F_1(0)$	0.63	0.74	0.76 ± 0.12	$0.64^{+0.08}_{-0.04}$
$F_2(0)$	0.32	-	0.38 ± 0.06	$0.32^{+0.04}_{-0.02}$
$F_3(0)$	0.21	-	0.26 ± 0.04	-

We have fitted our numerical results of the form factors in $B \rightarrow \pi, K, \rho, K^*$ into dipole forms and we have shown that our results agree well with those in the literature.

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Figure Captions

Figure 1: The values of F_T as a function of the momentum transfer p^2 for $B \rightarrow K$.

Figure 2: Same as Figure 1 but for $B \rightarrow \pi$.

Figure 3: The values of $F_{1,2,3}$ as a function of the momentum transfer p^2 for $B \rightarrow K^*$.

Figure 4: Same as Figure 3 but for $B \rightarrow \rho$.

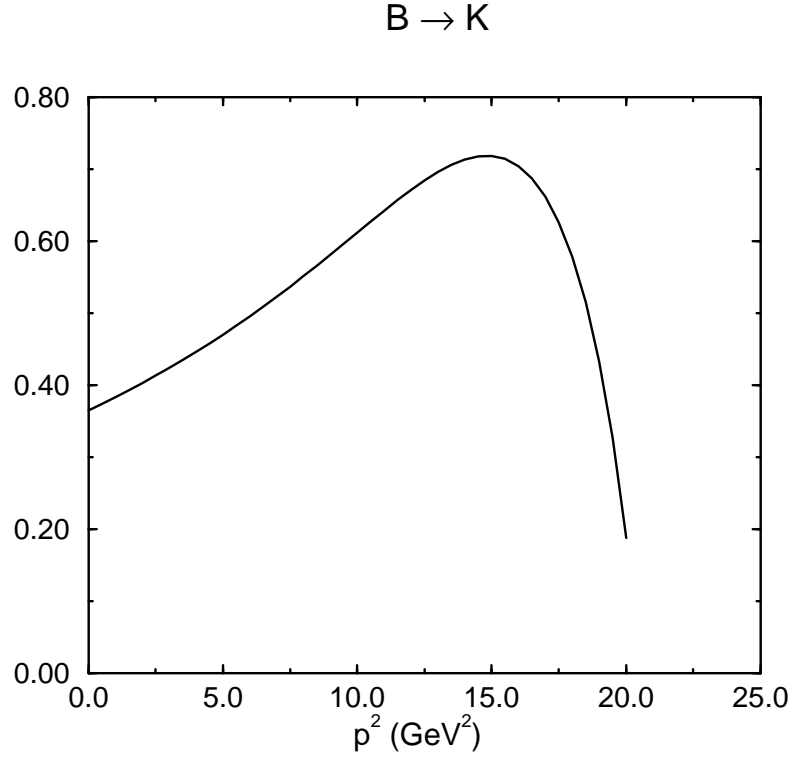


Figure 1: The values of the form factors F_T as functions of the momentum transfer p^2 for $B \rightarrow K$.

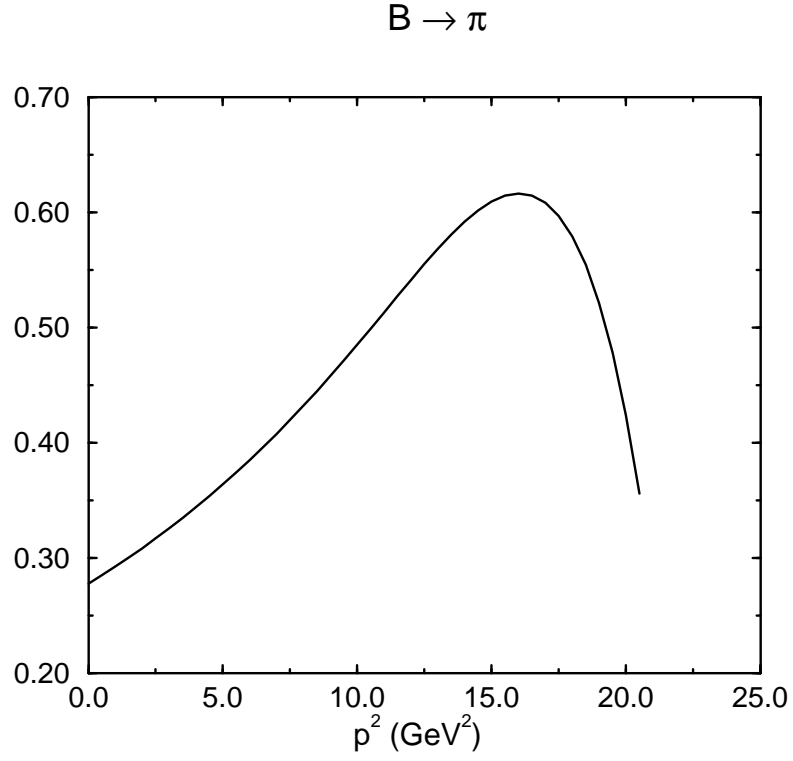


Figure 2: The values of the form factors F_T as functions of the momentum transfer p^2 for $B \rightarrow \pi$.

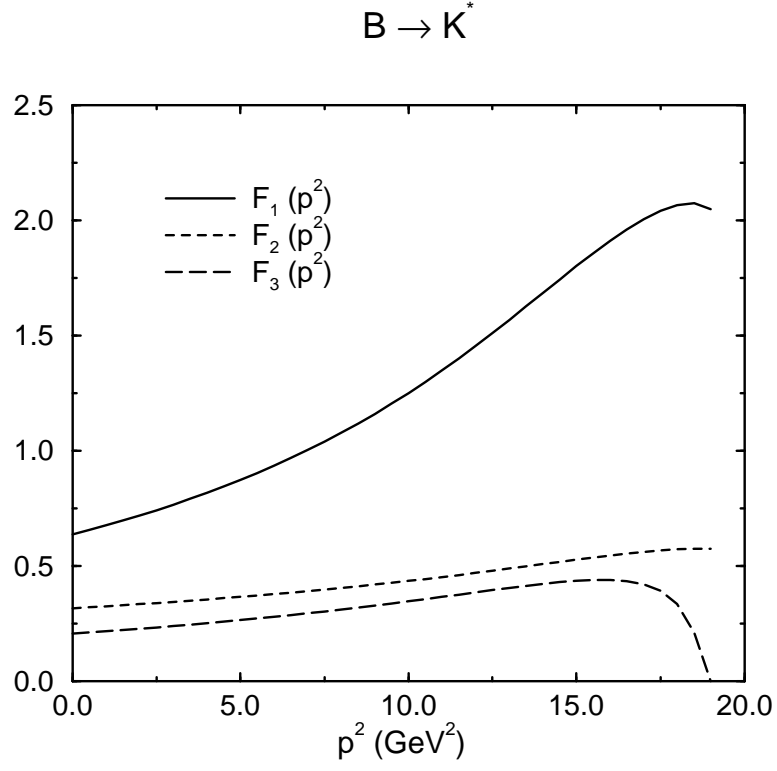


Figure 3: The values of the form factors $F_{1,2,3}$ as functions of the momentum transfer p^2 for $B \rightarrow K^*$.

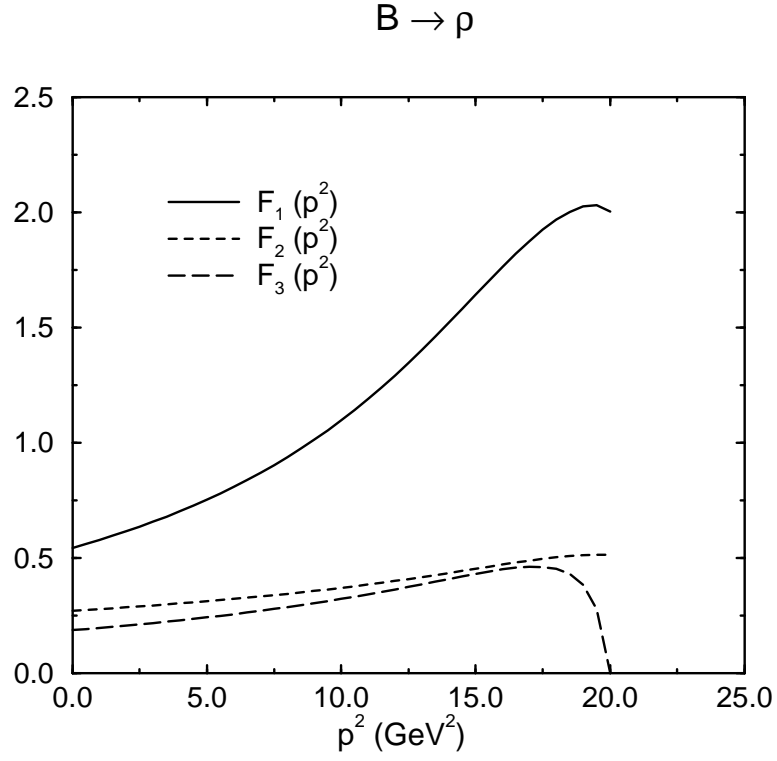


Figure 4: The values of the form factors $F_{1,2,3}$ as functions of the momentum transfer p^2 for $B \rightarrow \rho$.